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Analysis of Base Drag Reduction by Base and/or External Burning

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A simplified analysis that, however, treats all the primary physical and chemical processes of the flow has been developed. Numerical examples based on realistic values of the airstream and fuel conditions and parameters are used to illustrate the behavior of the resulting flow. It is shown that base burning at low fuel flow rates is attractive for base drag reduction. It is further shown that external burning adjacent to the base region is unattractive for base drag reduction or propulsion, either by itself or combined with base burning.

Nomenclature

b,c	= terms in Eq. (10)
A	= area of the viscous region
$\bar{A} \equiv A/A^*$	= dimensionless area
c_p	= specific heat
C_p	= pressure coefficient
$\text{cov}(q_i,q_j)$	= covariance of q_i and q_j
d	= base diameter
d_j	= diameter of the injector zone
f_s	= stoichiometric fuel-air ratio
h	= base radius
$\bar{h}_f$	= heating value
$\dot{m}$	= mass flow in the viscous region
M	= Mach number
p	= pressure
$q = \rho u^2/2$	= dynamic pressure
r	= radial coordinate
T_0	= total temperature
U,u	= axial velocity
V	= mean velocity in the viscous region
$W \equiv \dot{m}/\dot{m}^*$	= normalized mass flow
x	= axial coordinate
$\bar{x} \equiv x/\sqrt{A^*}$	= normalized axial coordinate
δ	= boundary-layer thickness
γ	= ratio of specific heats
ρ	= density
θ	= deflection angle
λ	= $\sqrt{M_I^2-1}$
ϕ	= factor in Eq. (9)

Subscripts

∞	= undisturbed external stream conditions
1	= external stream conditions behind the interaction shock
2	= external stream conditions just ahead of the base
b	= base values
e	= edge conditions
j	= initial conditions in the injectant

Superscript

$(^*)$	= viscous throat values

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Introduction

THE concepts of base injection and base injection with burning in an attempt to reduce the base drag, or even provide thrust, on supersonic projectiles has been of interest for some years. Much of the work through early 1974 is described in Ref. 1. In Ref. 2, a suggestion for locating the heat release zone adjacent to, but outside, the viscous base flow was presented with a view toward perhaps improving performance. One can also envision combined systems where part of the energy is released in the base flow region and part to the outside. Unfortunately, there have been no direct experimental and only limited analytical[3,4] comparisons of these various schemes in terms of performance. Indeed, unequivocal experiments of Strahle's "external burning" scheme either to prove or disprove the utility of the concept have not yet been performed.

The first base flow analyses that directly treated all of the important flow phenomena are given in Refs. 5-7. These are restricted to the planar geometry and they do not handle injection,[6] much less injection and burning. The removal of all of these restrictions is not trivial[5] and has not yet been accomplished.[4,8,9]

In Ref. 2, Strahle treated the planar, external only burning situation by modifying the analysis of Ref. 5. This was successful, since he did not treat injection or burning in the base region itself. This work represented an important first step in assessing a new concept in a preliminary way. However, several important phenomena were only crudely modeled. First, the mixing of freestream air with the fuel in the external flame was not directly treated. Second, the fuel injection process, including the unavoidable interaction shock(s) was not analyzed. Third, the influence of the cooling effect of the entrainment and mixing of the freestream air in the combustion zone was not considered. This can be very important. Fourth, the net heat release rates that were assumed for the examples presented turn out to be very, probably unattainably, high for known fuels. Last, it appears that the formulas used for the pressure rise coefficient are in error by a factor of 2.0 too high.

In Ref. 4, the analysis of Ref. 2 was extended to the axisymmetric geometry. However, most of the limitations mentioned above with regard to Ref. 2 were retained. An attempt was made to model heat release levels more realistically, but entrainment and mixing in the heat release zone was still not treated. The exclusion of injection and burning in the base region means that comparisons between external and base burning or combinations of the two cannot be made.

In Refs. 10 and 11 a simplified Crocco-Lees type of analysis for planar flows with base injection and burning was successfully developed. Agreement of the predictions with available experiments was shown to be good. The approach

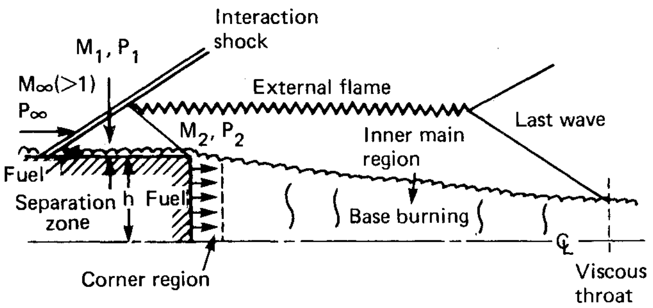

Fig. 1 Schematic of flowfield model with external burning.

involves separate models for separate regions: the main mixing region, the corner/base region, and a recirculation region for low injection rates. The models are then joined together to produce a complete, composite model of the flow. This was extended in Ref. 3 to include an external flame either with or without base injection and burning, still restricted to a planar geometry. The flow model is as shown in Fig. 1. The interaction shock and entrainment into the external flame are treated explicitly. The pressure perturbation produced by the external flame was modeled following the method of Ref. 12 and realistic heat release rates were used in the examples presented. Finally, the model of Refs. 9 and 10 was applied to axisymmetric flows with base injection, but not burning, in Ref. 13. The key additional item required was the development of a new, approximate pressure/flow-angle relation. Good agreement with experiments both with and without injection was achieved.

It is the purpose of this paper to present the extension of the axisymmetric analysis to cases with base and/or external burning. Representative numerical examples will be used to demonstrate several important, practical conclusions.

Analysis

The analytical models used for the various regions of the flow are presented separately below. Finally, the matching of the individual solutions to produce a complete, composite solution is discussed.

Inner Main Mixing Region

The largest part of the flow is called the "inner main mixing region" (see Fig. 1). From the viscous throat upstream to the boundary between this and the near-base region, the inner flow is treated as one-dimensional with entrainment from the outer stream. With the one-dimensional assumption, many of th equations are the same in the planar and axisymmetric geometries. We begin with the equations in Ref. 14 to determine the Mach number variation,

$$\frac{dM^2}{M^2} = -2\frac{[1+M^2(\gamma-1)/2]}{(1-M^2)}\left\{\frac{dA}{A} - \frac{(1+\gamma M^2)}{2}\frac{dT_0}{T_0} + \gamma M^2\left(\frac{U_e}{V}\frac{d\dot{m}}{\dot{m}}\right) - (1+\gamma M^2)\frac{d\dot{m}}{\dot{m}}\right\} \tag{1}$$

Here, we have taken the ratio of specific heats and the molecular weight of the fluid as constant, but these assumptions can be relaxed with only a small penalty in added complexity.

The term $(U_e/V)(d\dot{m}/\dot{m})$ is the momentum contribution of freestream fluid entrained into the mixing zone. Assuming $U_e \approx U_2$, the freestream velocity just upstream of the base, results in

$$\frac{U_e}{V}\frac{d\dot{m}}{\dot{m}} = \frac{M_2}{M}\left(\frac{T_{02}}{T_0}\right)^{1/2}\left[\frac{1+M^2(\gamma-1)/2}{1+M_2^2(\gamma-1)/2}\right]^{1/2} \tag{2}$$

Consider now the term, dT_0/T_0. Freestream air entrained into the mixing zone carries with it thermal energy based upon $T_{0,\infty}$ (note $T_{0,\infty} = T_{01} = T_{02}$). This can either heat or cool the mixing zone, depending on the relative magnitudes of $T_{0,\infty}$ and T_0. If there is base burning, the entrained air reacts with the fuel-rich mixture and releases heat in amount depending upon the heating value of the fuel. With a constant heating value assumption, these two effects may be represented as

$$\frac{dT_0}{T_0} = \left[\frac{T_{0,\infty}-T_0}{T_0} + \frac{\bar{h}_f f_s}{c_p}\right]\frac{d\dot{m}}{\dot{m}} \tag{3}$$

The quantity $(\bar{h}_f f_s/c_p)$ which reflects the heating value of the burning process will emerge as very important. Typical values range from about 2500 K (4500°R) for pure hydrocarbons down to 556 K (1000°R) or less for gas generator exhausts.

The behavior of the combustion process in the inner flame must also be characterized by another parameter. This is the dilution ratio where the flame is presumed to cease releasing heat as further air is entrained into the mixing region. This is an arbitrary input quantity in the present calculation; values in the range of 10/1 to 20/1 are typical.

The turbulent transport processes which appear here as entrainment, i.e., $d\dot{m}/dx$, can be modeled by extending the work of Ref. 4 as

$$\frac{d\dot{m}}{dx} = (0.01)2\pi r\rho_2 u_2\left|1-\left(\frac{\rho u}{\rho_2 u_2}\right)\right| \tag{4}$$

See Ref. 13 for a derivation.

Finally, the pressure variation in the inner flow, which will be balanced with that in the outer flow at the same x, is given by

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\dot{m}}\frac{d\dot{m}}{dx} - \frac{1}{A}\frac{dA}{dx} - \frac{1}{2M^2}\frac{dM^2}{dx}\left[\frac{1+M^2(\gamma-1)}{1+M^2(\gamma-1)/2}\right] + \frac{1}{2}\frac{1}{T_0}\frac{dT_0}{dx} \tag{5}$$

Pressure-Angle Relation

At this point, we had to confront (in Ref. 13) the problem of developing a pressure-angle relation for the outer flow in the axisymmetric situation. It should be emphasized that we would accept some crudity in order to retain the formulation of integrating upstream from the viscous throat. One can, in principle, start at the base and integrate down through the viscous throat, but the viscous throat represents a saddle-point singularity.[5] Only one set of upstream conditions, essentially the base pressure, will allow passage through this singularity mathematically, and carrying out this operation numerically is a difficult undertaking.[8,9,15] In Ref. 13, we developed an approximate form of the pressure coefficient relation from planar linearized supersonic theory which proved suitable for axisymmetric base flows

$$\frac{dp}{p} = \left(\frac{1.32}{\sqrt{3}}\right)\frac{\gamma M^2}{\sqrt{M^2-1}}d\theta \tag{6}$$

Except for the numerical factor, Eq. (6) is identical to the rigorously derived planar result, and thus it can play the same convenient role in our analysis. The adequacy of this approximation was demonstrated by comparisons with data for the pressure on boat-tailed bodies in Ref. 13.

Compatibility Condition

One must now match the pressure in the inner flow [Eq. (5)] to that in the outer flow [Eq. (6)]. An expression for θ is required, which can be obtained from the entrainment law [Eq. (4)]. The entrained fluid enters the mixing zone from the external flow because the flow angle θ is different than dr/dx.

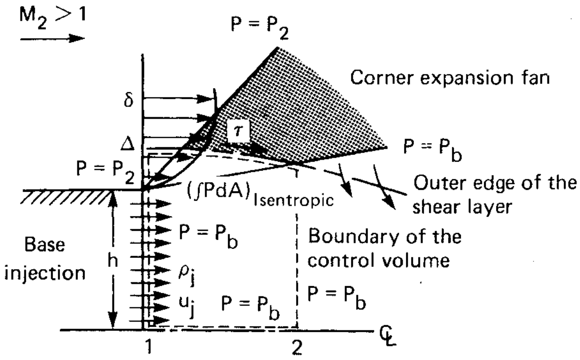

Fig. 2 Schematic of corner flow model.

Thus

$$\frac{\mathrm{d}\dot{m}}{\mathrm{d}x} = 2\pi r \rho_2 u_2 \left(\frac{\mathrm{d}r}{\mathrm{d}x} - \theta\right) \tag{7}$$

and

$$\theta = \frac{1}{2\sqrt{\pi A}}\frac{\mathrm{d}A}{\mathrm{d}x} - 0.01\left|1 - \frac{\rho u}{\rho_2 u_2}\right| \tag{8}$$

Using this in Eq. (6) and setting the result equal to Eq. (5) results in an expression that can be integrated with respect to x to give

$$\frac{\mathrm{d}\bar{A}}{\mathrm{d}\bar{x}} = (\bar{A})^{1/2}\Bigg[\left(\frac{\mathrm{d}\bar{A}}{\mathrm{d}\bar{x}}\right)^* + 2\sqrt{\pi}(0.01)\left[\left(\frac{\rho u}{\rho_2 u_2}\right)^* - \left(\frac{\rho u}{\rho_2 u_2}\right)\right]$$
$$- \frac{2\sqrt{\pi}\sqrt{M_2^2 - 1}}{\gamma M_2^2 \phi}\ell n\left(\frac{\bar{A}M\sqrt{\left(\frac{2}{\gamma+1}\right)\left(1 + \frac{\gamma - 1}{2}M^2\right)}}{W\sqrt{T_0/T_0^*}}\right)\Bigg] \tag{9}$$

where $\bar{A} \equiv A/A^*$, $\bar{x} \equiv x/\sqrt{A^*}$, $W = \dot{m}/\dot{m}^*$, and $\phi \equiv 1.32/\sqrt{3}$.

Mathematical Overview and Initial Conditions

The complete formulation is contained in Eqs. (1) [with (2)], (3), (4), and (9). They were all written in terms of the nondimensional variables and programmed for solution by means of the Hamming predictor/corrector method. The starting conditions in the mathematical analysis are the viscous throat conditions,

$$M = \bar{A} = W = T_0/T_0^* = 1$$

The solution would be straightforward except for the occurrence of the $(1 - M^2)$ term in the denominator of Eq. (1). This singularity is, however, the key to determining the base pressure, since only one value of the base pressure for a given problem will allow a smooth passage from subsonic to supersonic flow at the viscous throat. To find the starting conditions, it is necessary to apply L'Hospital's rule to Eq. (1). The result may be written schematically as

$$(\mathrm{d}M^2/\mathrm{d}\bar{x})^* = (-b + \sqrt{b^2 - 4c})/2 \tag{10}$$

where b and c are complex algebraic expressions, and we require $(\mathrm{d}M^2/\mathrm{d}\bar{x})^* > 0$ in order to have a passage from subsonic conditions in the base region to the supersonic downstream flow. The solution is carried upstream to match the solution for the corner region.

Corner Region

The corner region is of great importance, since it governs the initial conditions for the shear layer. A new model of the corner-flow problem was developed in Ref. 11 and extended to the axisymmetric geometry in Ref. 13. A schematic is shown in Fig. 2. It is presumed that some portion Δ of the body boundary-layer thickness δ participates in the formation of the shear layer. A control volume as indicated by the dotted line was considered and conservation of mass, momentum, and energy were applied. A double iterative process is used to determine a solution. After a problem is specified by given values of M_2, δ/h, and $\rho_j u_j$ and M_j, a series of trial values for p_b/p_2 is selected. For each trial value of p_b/p_2, a series of trial values for M_Δ is selected. For each case, a solution to the equations of motion is sought, which results in a series of possible solutions specified by pairs of values of $(\rho u)_{\mathrm{match}}$ and p_b/p_2. The correct solution is selected by matching with the rest of the flowfield solution as shown below.

Two assumptions were incorporated into the solution. First, the $\int P\mathrm{d}A$ term in the momentum equation was taken as that for an isentropic turn through the same pressure ratio, p_b/p_2. Also, the geometry of the system was obtained via the isentropic assumption. For the axisymmetric case here, the geometry was taken as if the flow along the outer edge of the control volume were planar. Second, the boundary-layer velocity profile was taken as a simple, one-seventh power law.

One further refinement in the treatment of the corner region is an attempt to account for the severely distorted profile shapes that occur for low base injection cases by introducing estimates of the "covariance" terms involved in the one-dimensional equations. The covariance of two quantities, q_i and q_j, is defined as

$$\mathrm{cov}(q_i, q_j) \equiv \overline{q_i q_j} - (\overline{q_i})(\overline{q_j})$$

where

$$\overline{q_i} \equiv \frac{1}{A}\int_0^A q_i(y)\,\mathrm{d}y, \quad \overline{q_i q_j} \equiv \frac{1}{A}\int_0^A q_i q_j \,\mathrm{d}y \tag{11}$$

When the variables are reasonably uniform across the transverse dimension of the flow, the covariance terms are very small and may be rejected, as was done implicitly in our treatment of the main mixing region. However, the reverse-flow profiles encountered, for example, in a recirculation zone render such an assumption invalid. By testing various assumed profile shapes, we have developed the following estimates for the covariance terms required in the analysis (see Refs. 11 and 13).

$$\mathrm{cov}(u,u)/u_2^2 = 0.05, \quad \mathrm{cov}(\rho,u)/\rho_2 u_2 = 0.02$$
$$\mathrm{cov}(\rho,u^2)/\rho_2 u_2^2 = 0.02, \quad \mathrm{cov}(\rho,T)/\rho_2 T_2 = 0.05 \tag{12}$$

Joining the Solutions for the Inner Flow

In order to obtain a complete solution for the inner flow, the solutions for the various inner regions must be joined. For planar flows in Refs. 10 and 11, a third region was introduced to connect the main mixing region and the corner region for cases where recirculation was presumed to exist. That additional region was dropped for axisymmetric cases in Ref. 13.

The complete solution to the inner flowfield is determined by matching the upstream marching viscous-inviscid interaction solution (for the inner main mixing region), which starts at the viscous throat with the corner region solution. The process is illustrated in detail in Refs. 11 and 13. A trace of $\rho u/\rho_2 u_2$ vs p_b/p_2 comes from the corner region solution. The one-dimensional, viscous-inviscid interaction solution also yields a trace of $\rho u/\rho_2 u_2$ vs p/p_2. The intersection represents the matching conditions for the problem.

The External Flame

Considering the most general problem, we can see that one must treat fuel injection and penetration, the interaction shock(s) produced by injection, air entrainment and mixing in the flame, and the pressure disturbances produced by the heat release zone. A simplified model that deals with most of these processes was developed in Ref. 3. Transverse fuel injection is assumed to produce an oblique interaction shock which compresses the air through a turn away from the body axis to an angle α. External burning then turns it back to the horizontal through another compression, and α is proportional to $\bar{h}_f f_s$, the product of the heating value of the fuel and its stoichiometric fuel-air ratio. It is implicitly assumed that the injection process is tailored to $\bar{h}_f f_s$ so as to produce a uniform, horizontal but compressed flow external to the base just ahead of the corner. This process can then be conveniently coupled to the inner region analysis if the external flame is taken to have a uniform heat release along its length. Thus, as far as the inner flow is concerned, the effect of the external flame is to reduce the edge Mach number and raise the "outer" stream static pressure.

From Ref. 12 we may write the pressure change due to a uniform heat release sheet as

$$C_p = \frac{(\gamma - 1)q}{\rho_2 u_2 (\gamma R T_2)\sqrt{M_2^2 - 1}} \frac{\sin\mu}{\sin(\mu + \alpha)} \tag{13}$$

where μ is the local Mach angle and q is a heat rate per unit time and area. It remains now to calculate the net heat release that results from the combustion of the air entrained into the fuel-rich external mixing zone. This entrainment rate can be estimated as for the inner mixing region, noting that there are two sides to the mixing region here. Thus,

$$\frac{d\dot{m}}{dx} = 2(0.01)2\pi r \rho_2 u_2 \left(1 - \frac{\rho u}{\rho_2 u_2}\right) \simeq 2(0.01)2\pi h \rho_2 u_2 \tag{14}$$

Again following our work for the inner region, the entrained air has an effective heating value when combined with the fluid in the fuel-rich zone, so that

$$q = c_p \dot{m}_{\text{entr}} dT_0 = d\dot{m}[\bar{h}_f f_s + c_p(T_{0,\infty} - T_0)] \tag{15}$$

If the first term in the brackets is much larger than the second, or both terms are combined into a "net" heating value $(\bar{h}_f f_s)_{\text{net}}$, then

$$q = \frac{\dot{m}_{\text{air}}(\bar{h}_f f_s)_{\text{net}}}{2\pi h \ell} \tag{16}$$

where ℓ is the total length of the external flame. Finally,

$$C_p = \frac{(\gamma - 1)2(0.01)(\bar{h}_f f_s)_{\text{net}}}{(\gamma R T_2)\sqrt{M_2^2 - 1}} \frac{\sin\mu}{\sin(\mu + \alpha)} \tag{17}$$

The minimum amount of fuel required to produce this situation can be back calculated as follows: First, ℓ is found from the base-flow analysis such that the last wave from the external burning region strikes the base flow at the viscous throat. The air entrained in this length equals

$$\dot{m}_{\text{air}} \approx 2(0.01)2\pi h \rho_2 u_2 \ell \tag{18}$$

so that the fuel required such that the flame is just stoichiometric at the end is

$$\dot{m}_{\text{fuel}}/(\rho_2 u_2 \pi h^2) = \dot{m}_{\text{air}} f_s/(\rho_2 u_2 \pi h^2) \tag{19}$$

It is useful to consider the energy equation (15) as applied to an external flame in order to determine the practical requirements to produce a given level of performance. Now, what is $(\bar{h}_f f_s)_{\text{net}}$? This is the "net" heat release due to air entrainment and combustion with the fuel. We can see from Eq. (15) that entrainment of cool (with respect to the flame) freestream air reduces the total effect of the heat release due to burning. For the external flame viewed as a whole, we can say

$$(\bar{h}_f f_s)_{\text{net}} \approx (\bar{h}_f f_s) + c_p[T_{0,\infty} - (T_0)_{\text{flame av}}] \tag{20}$$

The average temperature in the flame. $(T_0)_{\text{flame av}}$, can be approximated as

$$(T_0)_{\text{flame av}} \approx \frac{T_{0j} + (T_0)_{\text{flame final}}}{2} \tag{21}$$

The ideal practical arrangement is to have the external flame be stoichiometric at the downstream station where it influences the base region just at the viscous thrust. Approximating Eq. (15) for the external flame as

$$\Delta T_0 = T_{0_{\text{flame final}}} - T_{0j} \approx \left(\frac{\dot{m}_{\text{final}} - \dot{m}_j}{\dot{m}_{\text{av}}}\right) \times \left[\left(\frac{\bar{h}_f f_s}{c_p}\right) + (T_{0,\infty} - (T_0)_{\text{flame av}})\right] \tag{22}$$

we can close the system.

The whole process and some important conclusions can be illustrated with a representative example. Consider $M_\infty = 2.2$ and $T_{0,\infty} = 556$ K (1000°R). In order to produce $p_2/p_\infty = 1.10$, a modest goal, we can have an oblique shock and a compression produced by the flame each corresponding to a turn through only 0.75 deg. This reduces $M_\infty = 2.2$ to $M_1 = 2.17$ and $M_2 = 2.14$. Using Eq. (17) with $c_p = 0.24$, we get a required value of $(\bar{h}_f f_s/c_p)_{\text{net}} = 409$ K (736°R). If we take $T_{0j} = 1389$ K (2500°R), and $(\dot{m}_{\text{air}}/\dot{m}_{\text{fuel}})_{\text{stoich}} = 5$, Eqs. (20-22) give a required value for $(\bar{h}_f f_s)/c_p) = 1550$ K (2790°R). Results of this type over a range of the parameters involved are shown in Fig. 3. Two important conclusions can be drawn: 1) it is quite difficult to produce large pressure rises via heat release in an unconfined supersonic flow; and 2) the higher the value of T_{0j} (as might be attractive for ignition), the higher the required heating value of the fuel to produce a given pressure rise. This is opposite to the natural course of things for gas generator exhausts.

Results

A representative case has been selected for detailed analysis, so that actual performance levels can be predicted and studied and so that direct comparisons between base-only

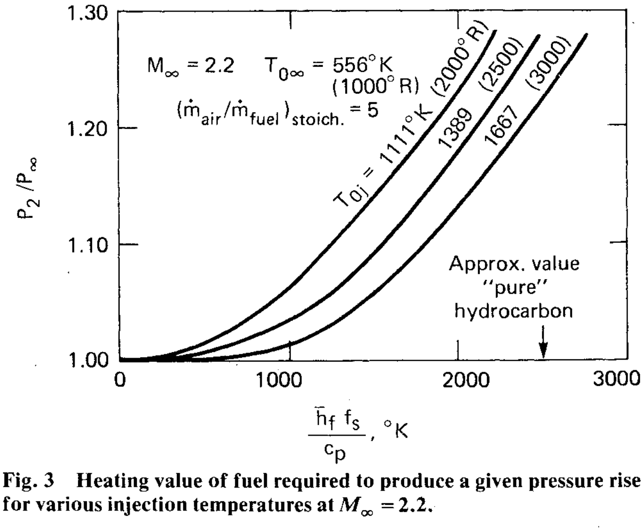

Fig. 3 Heating value of fuel required to produce a given pressure rise for various injection temperatures at $M_\infty = 2.2$.

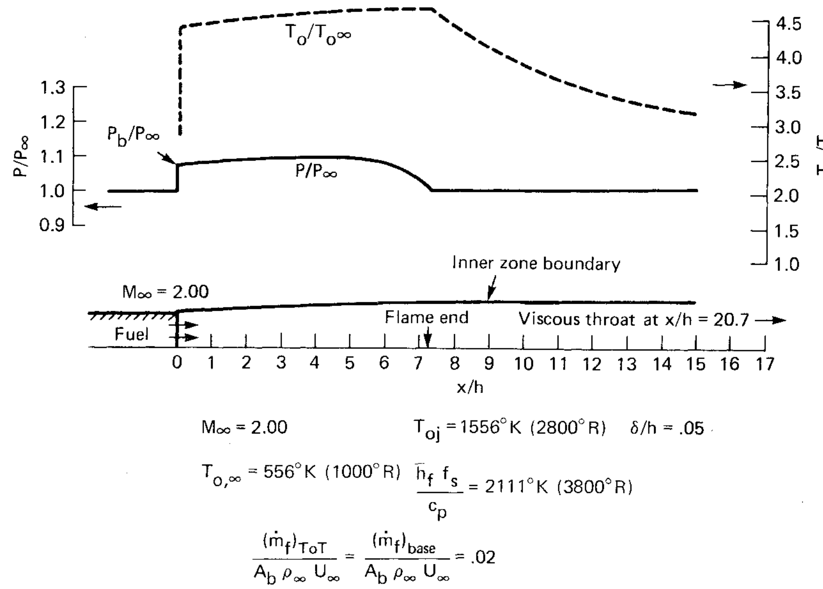

Fig. 4 Flowfield prediction for typical base burning case.

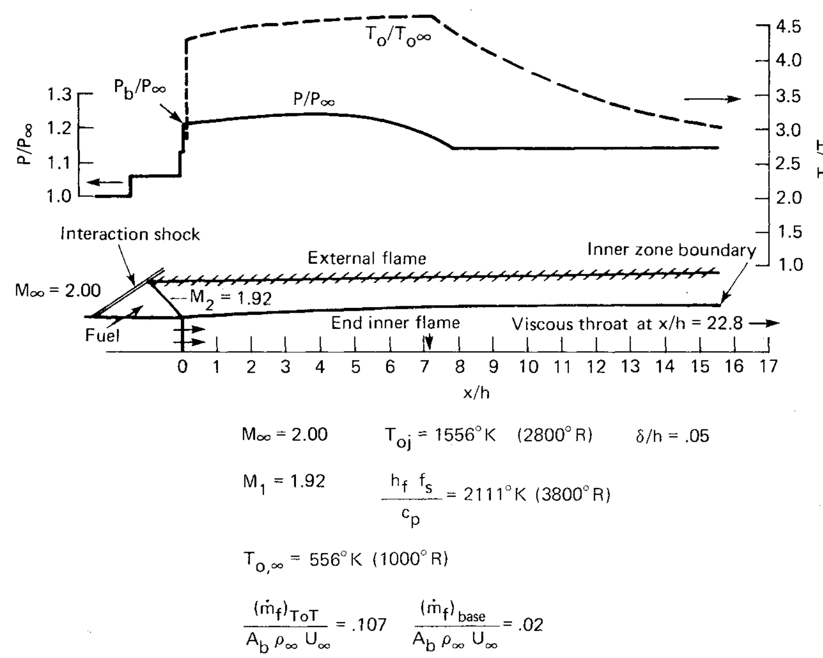

Fig. 5 Flowfield prediction for typical combined base/external burning case.

burning and a base/external burning combination can be made. We have chosen $M_\infty = 2.0$ and $T_{0,\infty} = 556$ K (1000°R) and a ratio of boundary-layer thickness to base radius of $\delta/h = 0.05$. A fuel with favorable performance potential was postulated. With $T_{0j} = 1556$ K (2800°R), we have taken $\bar{h}_f f_s / c_p = 2111$ K (3800°R). These values represent a tradeoff between an elevated temperature level to aid ignition and sufficient heat release capability to be attractive as an external burning fuel. The fuel was also assumed to burn freely with entrained air up to $\dot m_{\rm air}/\dot m_{\rm fuel} = 10.5$, at which point it would extinguish. In all of this, "fuel" is taken to mean the fluid injected into the main airstream about the body, whether or not an internal gas generator is employed.

The results for a base-only burning configuration with $\dot m_{\rm fuel}/A_b \rho_\infty u_\infty = 0.02$ are shown in Fig. 4. It can be seen that a base pressure to freestream pressure ratio, $p_b/p_\infty = 1.075$, is predicted. This is to be compared with a value of 0.56 to be expected for no base injection or combustion. The base region is quite long, some 21 base radii, and the heat release ceases at $x/h \approx 7.2$. Also, the outer edge of the mixing region grows slowly to a value about 40% greater than the base radius by the viscous throat.

Comparable results for a combined base/external burning case at the same conditions and with the same fuel are shown in Fig. 5. The assumed fuel is capable of producing a 1 deg turn at these conditions with an external flame. With our flow model, this results in $M_1 = 1.96$ and $p_1/p_\infty = 1.06$ and $M_2 = 1.92$ and $p_2/p_\infty = 1.13$. With $(\dot m_{\rm fuel})_{\rm base}/A_b \rho_\infty u_\infty = 0.02$, we predict $p_b/p_\infty = 1.21$. However, a long external flame is required to keep a constant Mach number and an elevated pressure between the outer flames and the inner base region. For this case, 4.33 times as much fuel is required for the external flame than for the inner region. Thus, while the base pressure level achieved is significantly higher than for base-only burning, the increase in cost in terms of fuel flow required is higher still. Similar results have been found for all cases studied, both in the planar geometry[3] and the axisymmetric cases in the present investigation.

Conclusion

A comprehensive, but approximate, analysis has been developed that deals in a realistic way with the main physical and chemical processes of importance in base, external, or combined base/external burning on axisymmetric, supersonic bodies. Careful consideration has been given to the effects of the values of fuel and airstream conditions and parameters in ranges of practical interest. Numerical examples using representative values were used to illustrate the major effects. Several conclusions can be drawn from these results. First, base burning with low fuel flow rates is attractive for base drag reduction. Second, the negative influence of entraining cool, freestream air into a hot flame zone on the "net" heating to be realized is important and must be treated in any analysis. The injection of hot "fuel" in an attempt to aid ignition makes this situation worse. Third, high heating value "fuel" is required if any appreciable pressure rise is to be gained from external burning. These last two points tend to make the use of a solid-fuel-fired gas generator unattractive for such applications. Finally, external burning either by itself or combined with base burning is not attractive for base drag reduction or propulsion from a fuel usage point of view.

One can envision some further extensions to this work including, for example, the influence of spin and real gas effects. This should not be difficult. However, the overall conclusions reached here would not be materially changed by the addition of any such effects.

Acknowledgment

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References

[1] Murthy, S.N.B. (Ed.), *Progress in Astronautics and Aeronautics: Aerodynamics of Base Combustion*, Vol. 40, AIAA, New York, 1976.

[2] Strahle, W. C., "Theoretical Considerations of Combustion Effects on Base Pressure in Supersonic Flight," *12th Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, Pa., 1968.

[3] Schetz, J. A. and Favin, S., "Analytical Comparison of the Performance of Different Base Burning Modes," *AIAA Journal*, Vol. 14, Sept. 1976, pp. 1337-1338.

[4] Mehta, G. K. and Strahle, W. C., "Analysis of Axially Symmetric External Burning Propulsion for Bluff-Base Bodies," *AIAA Journal*, Vol. 17, March 1979, pp. 269-270.

[5] Crocco, L. and Lees, L., "A Mixing Theory for the Interaction between Dissipative Flows and Nearly Isentropic Streams," *Journal of Aeronautical Sciences*, Vol. 19, Oct. 1952, pp. 649-676.

[6] Alber, I. E., "Integral Theory for Turbulent Base Flows at Subsonic and Supersonic Speeds," Ph.D. Thesis, California Institute of Technology, Pasadena, Calif., June 1967.

[7] Alber, I. E. and Lees, L., "Integral Theory for Supersonic Turbulent Base Flows," *AIAA Journal*, Vol. 6, July 1968, pp. 1343-1351.

[8] Smithey, W. and Fuhs, A. E., "Base Pressure Calculations with External Burning," *Progress in Astronautics and Aeronautics: Aerodynamics of Base Combustion*, Vol. 40, edited by S.N.B. Murthy, AIAA, New York, 1976, pp. 407-424.

[9] Strahle, W. C., Mehta, G., Hubbartt, J. E., "Progress on a Base Flow Model for External Burning Propulsion," *Progress in Astronautics and Aeronautics: Aerodynamics of Base Combustion*, Vol. 40, edited by S.N.B. Murthy, AIAA, New York, 1976, pp. 339-348.

[10] Schetz, J. A. and Billig, F. S., "Approximate Analysis of Base Burning in Supersonic Flow," *Progress in Astronautics and Aeronautics: Aerodynamics of Base Combustion*, Vol. 40, edited by S.N.B. Murthy, AIAA, New York, 1976, pp. 385-405.

[11] Schetz, J. A. and Billig, F. S., "Simplified Analysis of Supersonic Base Flows Including Injection and Combustion," *AIAA Journal*, Vol. 14, Jan. 1976, pp. 7-8.

[12] Tsien, H. S. and Beilock, M., "Heat Source in a Uniform Flow," *Journal of Aeronautical Sciences*, Vol. 16, Dec. 1949, p. 756.

[13] Schetz, J. A., Billig, F. S. and Favin, S., "Approximate Analysis of Axisymmetric Base Flows with Injection in Supersonic Flow," *AIAA Journal*, Vol. 18, Aug. 1980, pp. 867-868.

[14] Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Ronald Press, New York, 1953, Chap. 8.

[15] Peters, C. E., Giel, T. V., and Phares, W. J., "Analytical Study of External-Burning Propulsion," AEDC-TR-77-1, April 1977.